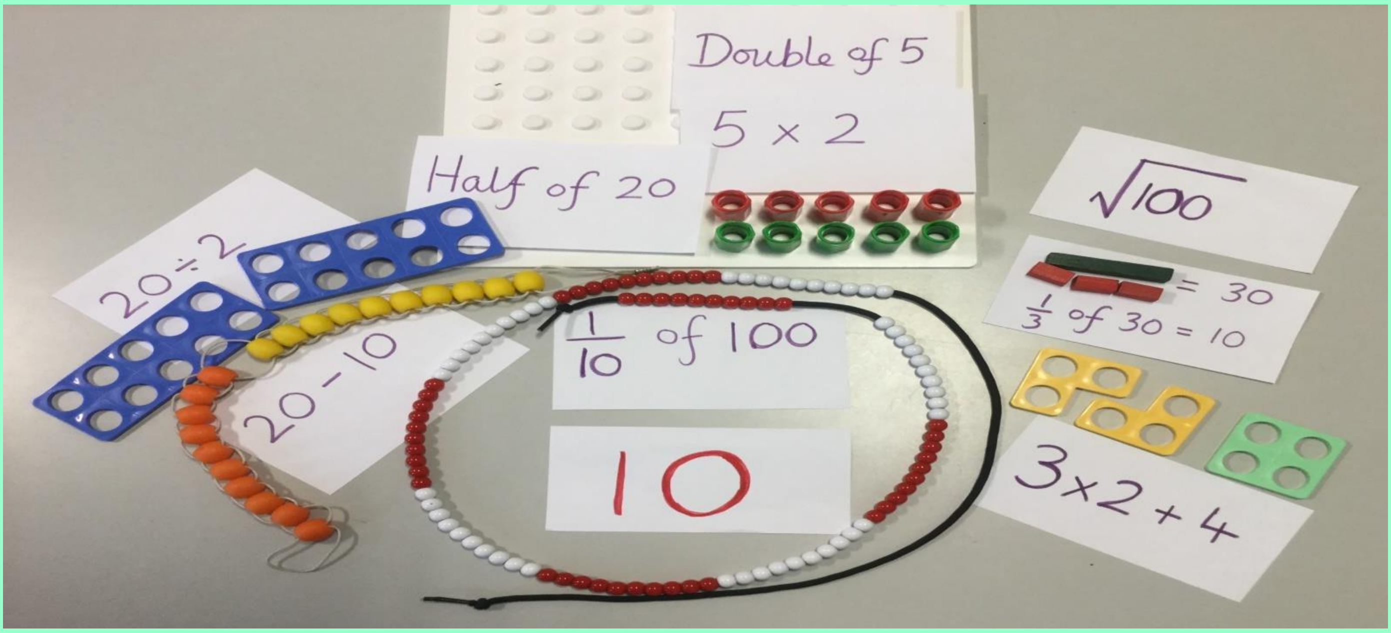
DMA CLUSTER CALCULATIONS POLICY



Downham Market Academy
Aspire Ambition Achieve

The Bridges Federation
Working together for success

Watlington C P School

HILLCREST

Nelson Academy

Southery Academy

Shouldham

Riverside Federation

DENVER V.C.P.

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Schools involved

This calculations policy has been developed and written by the Maths Subject Leaders from the following schools of the Downham Market cluster, led by Samina Asif. It aims to aid transition by providing a consistent approach to the written and mental calculation methods. Guidance was taken from 'The Calculation Guidance: NCETM October 2015' and Alison Borthwick, Norfolk Maths Advisor.

Name of school	**Name of Maths Subject Leaders involved in the process**
Denver VC C of E Primary School	Linda Pennock
Downham Market Academy	Darielle Pleterski
Hillcrest Primary School	Lisa Coe
Hillcrest Primary School	Robert Brown
Nelson Academy	Davina Nyarko
Southery Academy	Jonathan Frost
St Martin At Shouldham C of E Primary Academy	Louise Waring
Ten Mile Bank Riverside Academy and Hilgay Riverside Academy	Sarah Turner
The Bridges Federation Wimbotsham and Stow Community School and Magdalen Village School	Michelle Harding
Watlington Community Primary School	Samina Asif

RATIONALE

In June 2015, the participating schools conducted a calculations skills audit (taken from Borthwick and Harcourt-Heath 2008) to not only find out what strategies were being widely used but also to examine their impact on the outcomes.

The results of correct calculations from these schools were collated as follows:

Year group	Addition	Subtraction	Multiplication	Division
EYFS	88%	54%	52%	42%
Year 1	83%	49%	54%	43%
Year 2	76%	58%	73%	65%
Year 3	85%	61%	54%	51%
Year 4	89%	52%	44%	33%
Year 5	83%	74%	51%	45%
Year 6	93%	82%	36%	59%

Children in EYFS and Y1 often gave an answer without recording anything. There were only a few pupils, who had drawn pictures to represent their mathematical thinking. The greatest difference observed, between the application of strategies in KS1 and KS2, was the lack of concrete or pictorial representations in KS2. Generally, the inability to make sensible estimates and check the reasonability of answers suggested poor number sense across the key stages. Counting errors were very common in KS1 whereas in KS2, errors detected were mostly where formal column methods were used without an understanding. The most successful strategies used were **number lines for subtraction** and **grid method for multiplication**. Number lines, as proven by research, show a natural progression of number and are applicable to all areas of the calculation. However, as agreed by the Subject Leaders, more formal methods only need to be taught ***when pupils are ready.***

Pupils in KS3 somehow feel it is 'uncool' to show their work on number lines and end up being too hasty in using formal methods without an understanding. Some pupils had no strategies for multiplication and division. Pupils from Years 7 and 8 with a range of abilities were given a numeracy skills audit in June 2015. Pupils in year 7 were given the Year 5 skills test and the Year 8 pupils were given the Year 6 skills test on calculations.

The overall results are given below:

Year group	Addition	Subtraction	Multiplication	Division
Year 7	92%	73%	64%	68%
Year 8	82%	47%	21%	22%

The most common methods used were the column method for addition and subtraction, the grid method for multiplication and a combination of short division and chunking for division. The skills audit clearly showed that while the column method generally worked for addition, it was not so successful for subtraction. The grid method for multiplication was more successful with the Year 7 skills audit compared to the Year 8 skills audit. Pupils were less successful where division involved remainders.

Aims and Principles

The main aims of the calculations policy are to:

- Create love and enthusiasm for Maths
- Develop a deeper understanding of calculations
- Improve pupils' ability to calculate accurately
- Ensure a consistent approach within schools and across the cluster
- Prepare pupils for the next key stage

As part of the **monitoring** programme, Subject Leaders will carry out an internal scrutiny of books, pupil interviews and an annual calculations skills audit. This will help to evaluate the impact of the policy during the annual review at the end of each academic year.

Addition

Definition: Addition is finding or calculating the total of two or more sets, numbers or amounts. It is the inverse of subtraction.

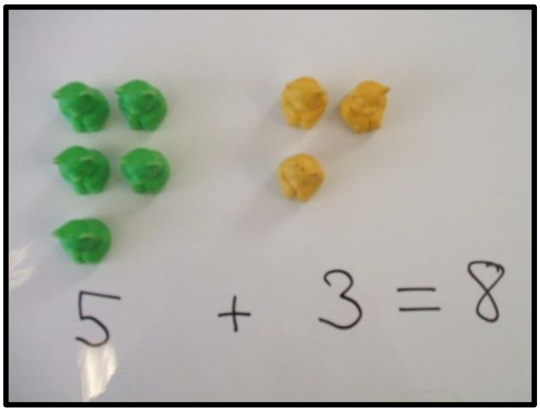
Core skills

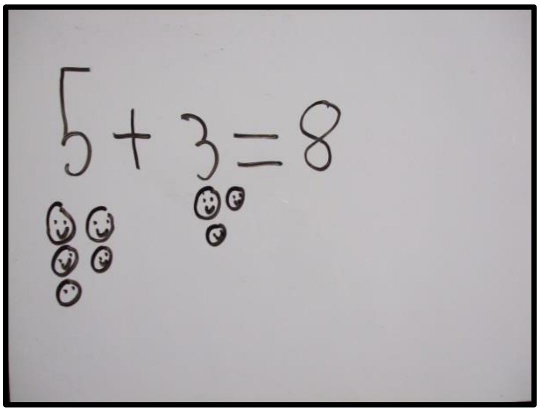
To progress and use efficient methods of addition, pupils will need to be able to:

- Use one to one correspondence
- Count forwards and backwards
- Understand that addition is commutative
- Partition
- Have rapid recall of number bonds
- Understand addition as the inverse of subtraction.

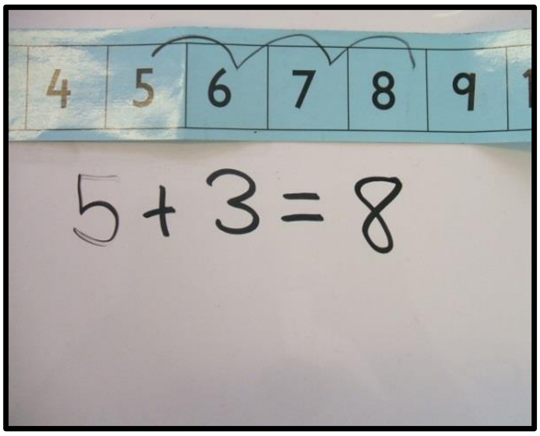
Early learning

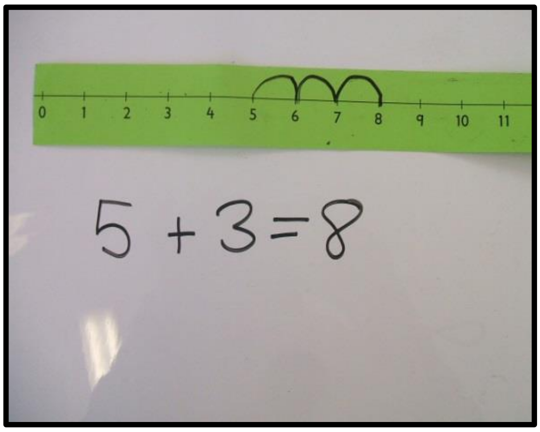
Pupils use concrete and pictorial methods for calculating.





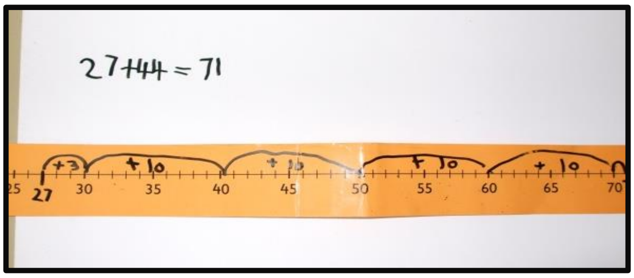
Using the core skill of counting forwards, pupils use number tracks and marked number lines to find the total. This can be shown with a bar model as well as using concrete resources.

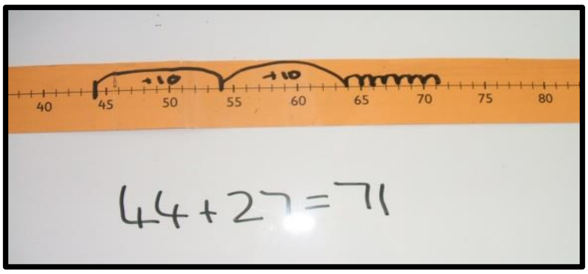


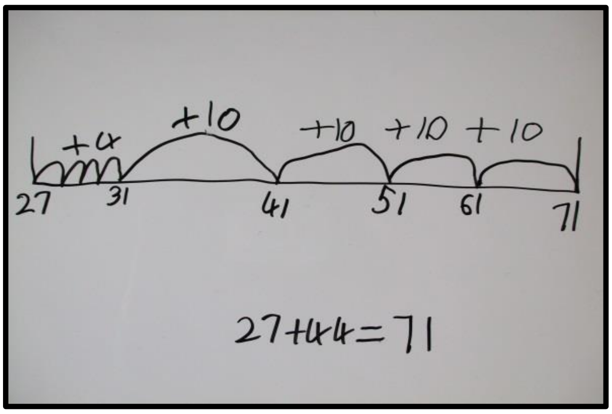


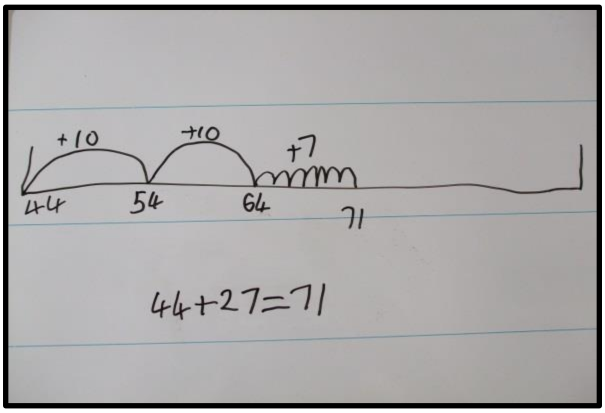
Written methods

Counting on an unmarked number line using various efficient methods



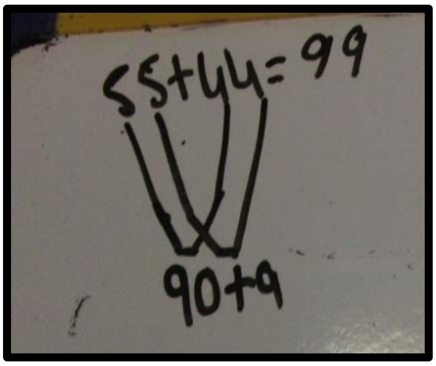


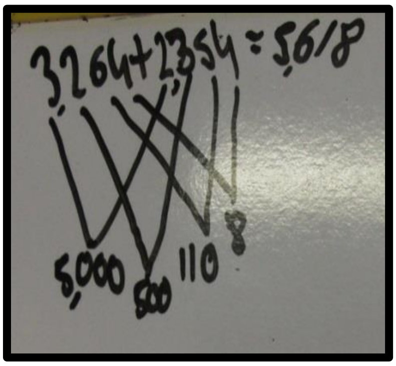


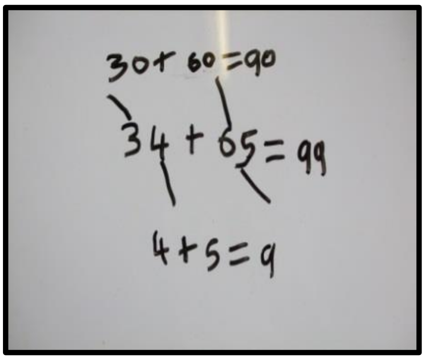


Progress to Partitioning

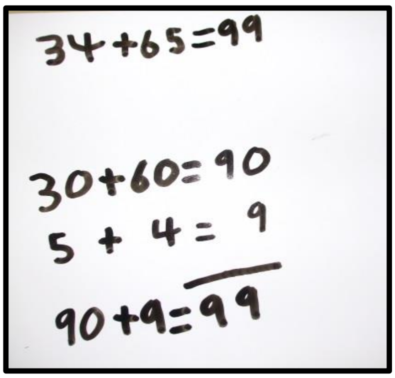
Partition and add horizontally:

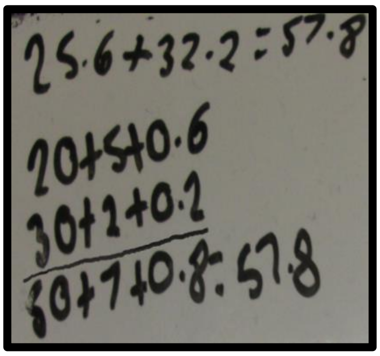


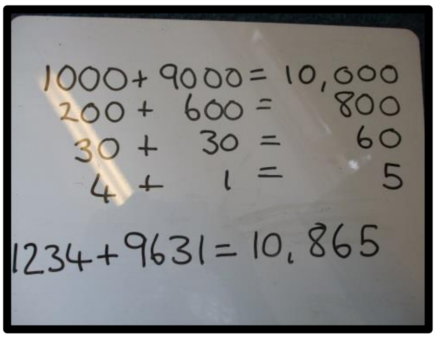




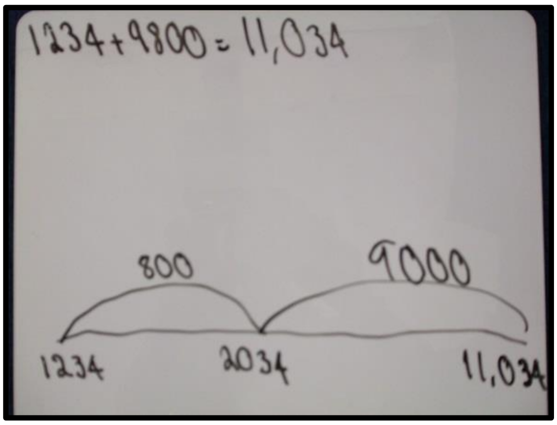
Partition horizontally and add vertically:

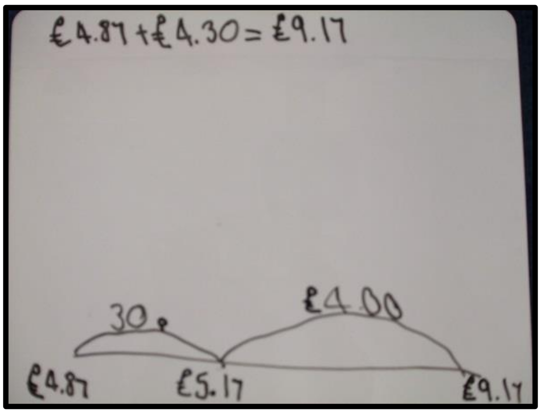




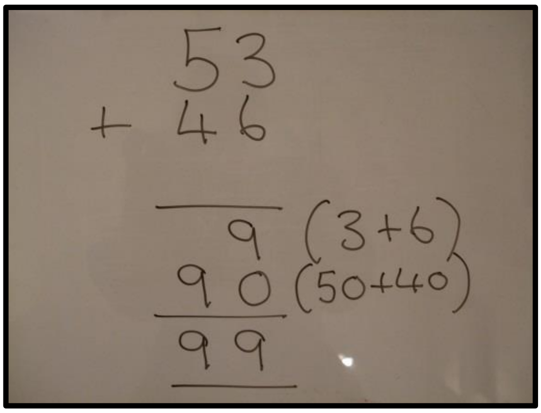


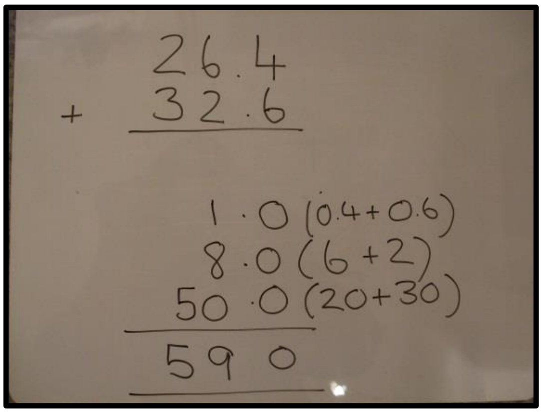
Add on an empty number line:



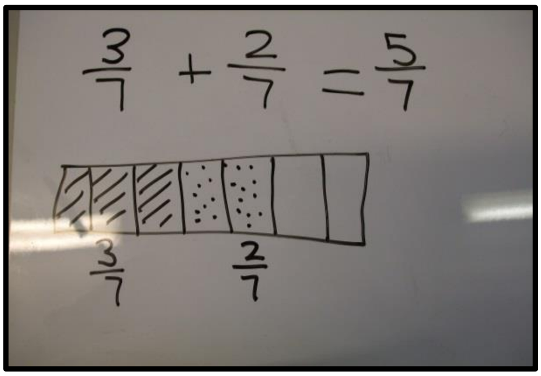


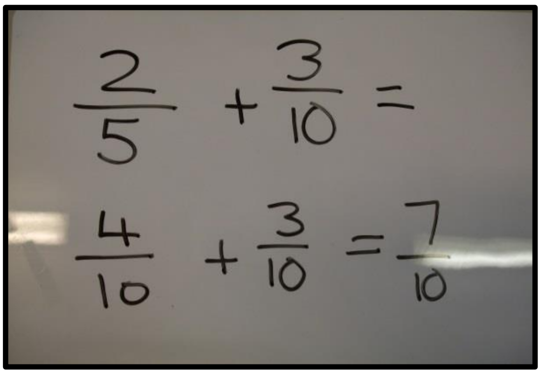
Pupils continue to partition and move towards vertical methods to increase efficiency and fluency.



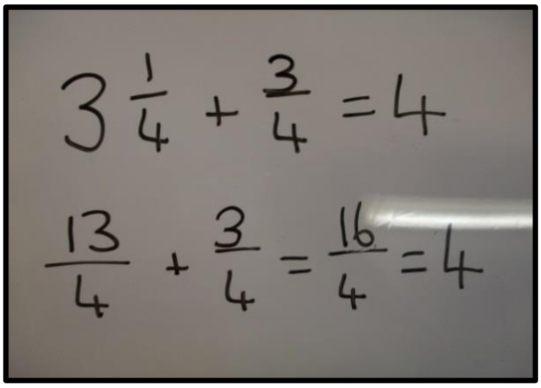


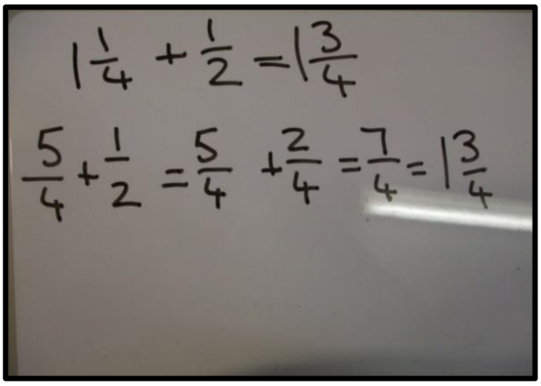
Adding fractions with a common denominator and different denominators:





Adding mixed numbers with a common denominator and different denominators:



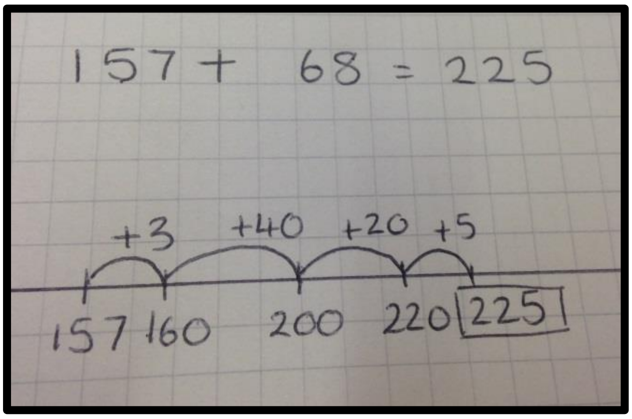


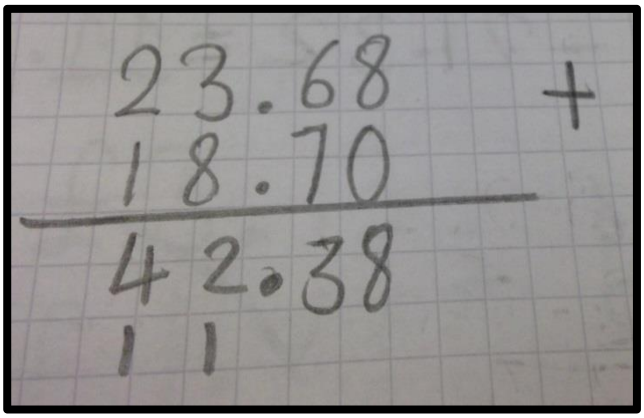
Formal written method

The formal method should only be taught when appropriate. Further examples can be found in the appendix.

Expectations of Downham Market Academy

Pupils continue to use mental methods, number lines and progress to column methods with a greater understanding.





Web Links: https://www.ncetm.org.uk/

Subtraction

Definition: Subtraction is the taking away of one number or amount from another, or finding the difference between two numbers or amounts.

Core Skills:

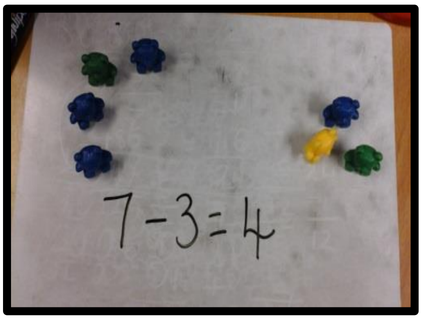
To progress and use efficient methods of subtraction, pupils will need to be able to:

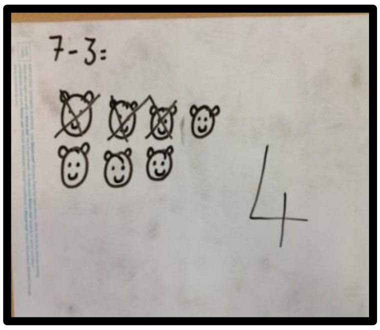
- Count forwards and backwards
- Understand that subtraction is not commutative
- Partition numbers
- Rapidly recall number bonds
- Understand that subtraction is the inverse of addition
- Recognise the number to be subtracted.

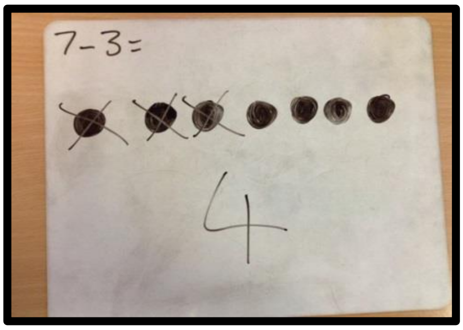
Early Learning

Children are encouraged to use a range of concrete resources. In subtraction, resources are physically taken away. They then develop ways of recording calculations using pictures, crossing out these images to represent subtraction. These pictures may begin as representations of the problem but will develop to more informal representations.

e.g. 7 bears are having a party. 3 bears leave. How many bears are left at the party?



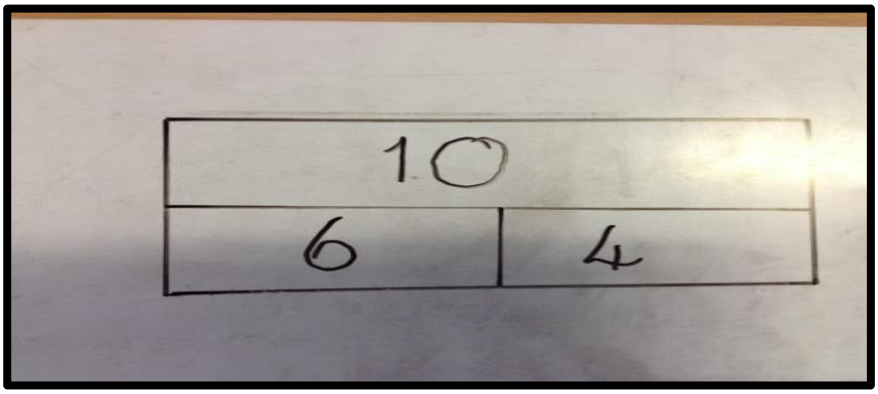




Bar Model

A further method for more abstract pictorial representation is the bar model, which supports children to visualise the part-part-whole model.

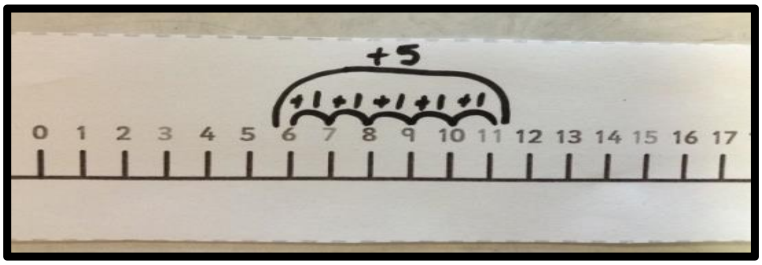
e.g. 6 + 4 = 10



Number Tracks and Number Lines

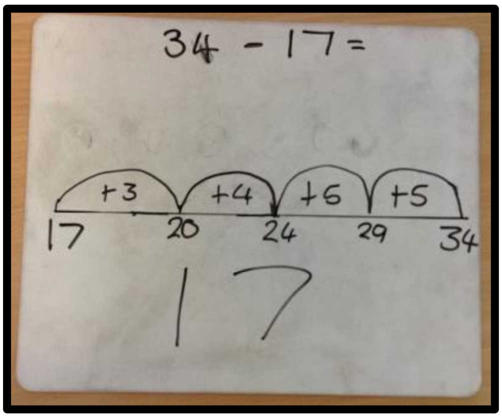
Use marked number lines alongside concrete objects and pictorial representations. Include both counting up to find the difference and counting back to subtract.

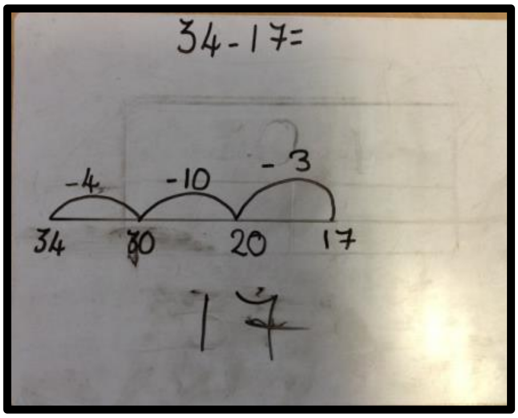
e.g. Counting up on a marked number line to show 11 – 5 = 6



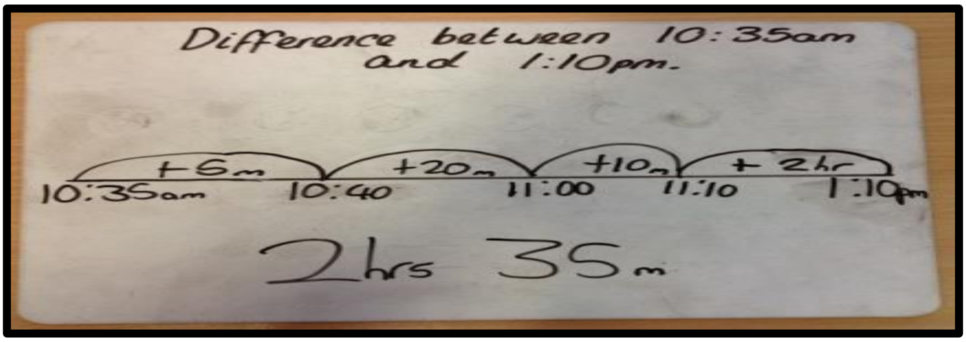
Written Methods – 2, 3 and 4 Digits

Use number lines to count on or back in ones to begin with, and develop more efficient jumps. As understanding increases, progress to empty number lines.



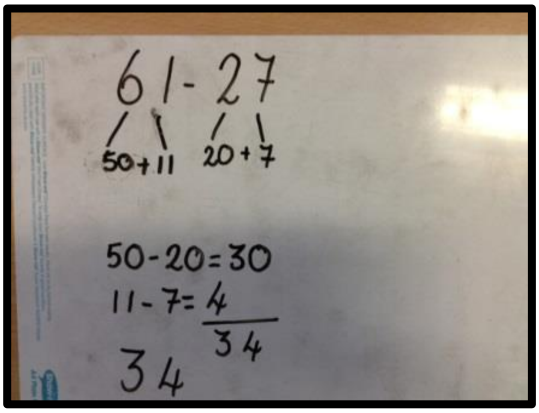


Number lines can also be used for calculating time differences:

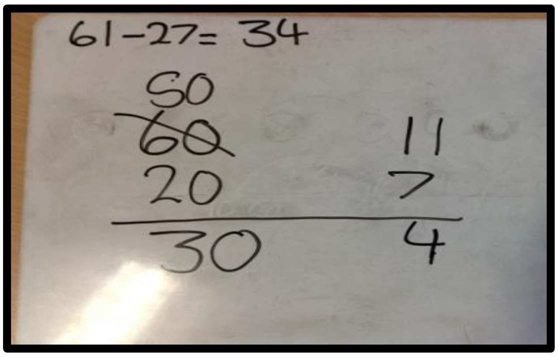


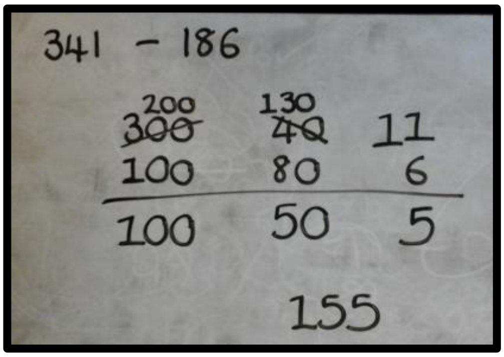
Progress to partitioning:

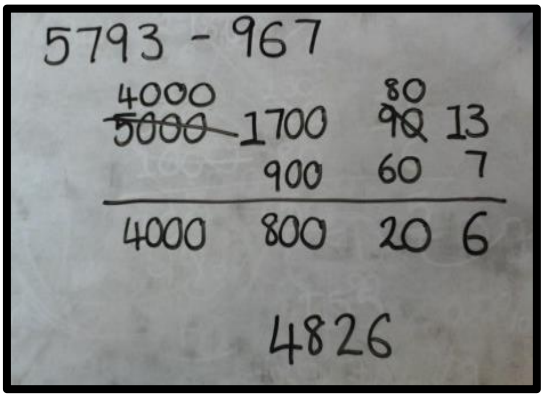
Initially, the children will be taught using examples where exchange is not required. They begin with partitioning the numbers and subtracting the tens and ones separately before recording underneath:

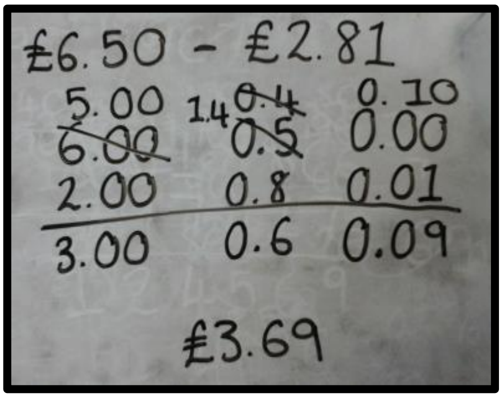


Begin with examples where 'exchanging' is necessary and encourage partitioning of the number in different ways. This method should be taught in a range of contexts to increase understanding, including larger numbers.

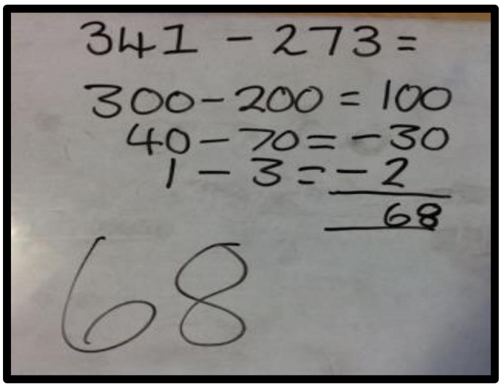






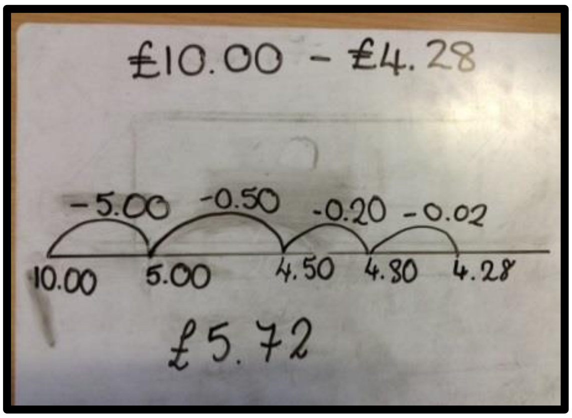


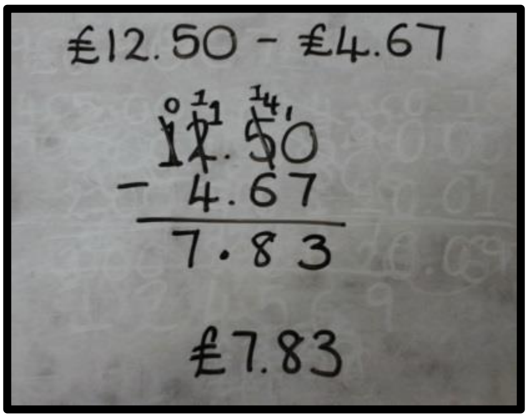
Develop an understanding of subtracting negative numbers:



Decimals

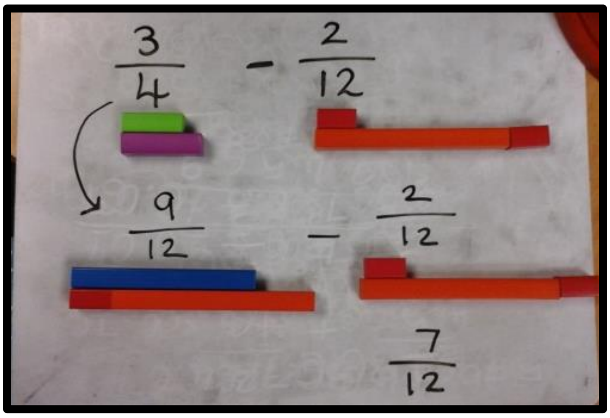
All written methods can be applied to different contexts, such as decimal subtraction.

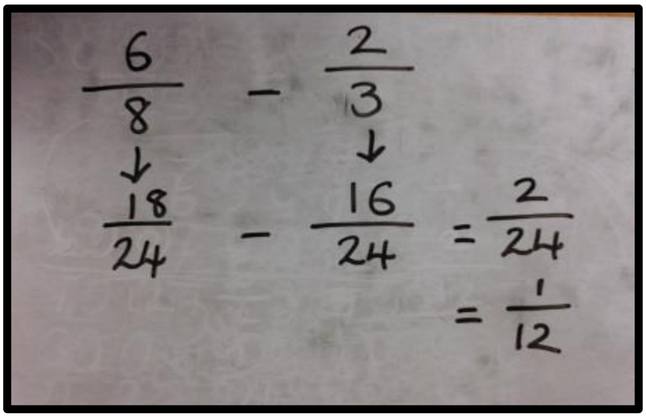




Fractions

Use concrete and pictorial methods along with abstract to subtract fractions.





Formal written method

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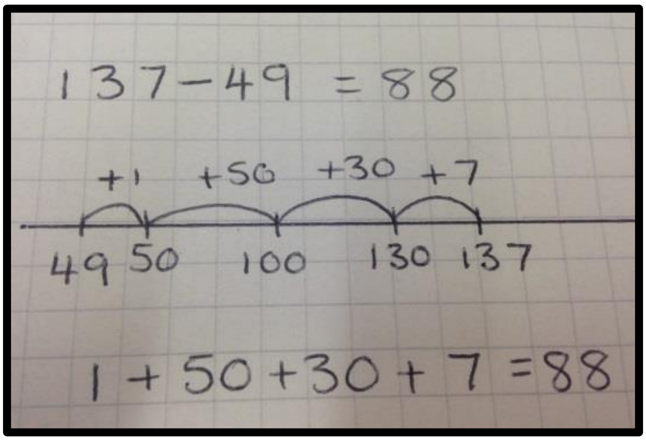

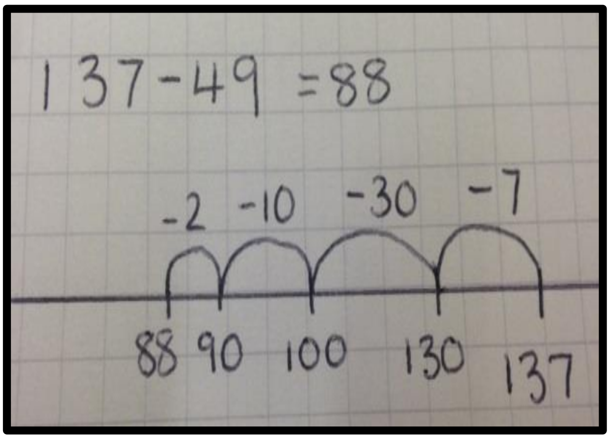

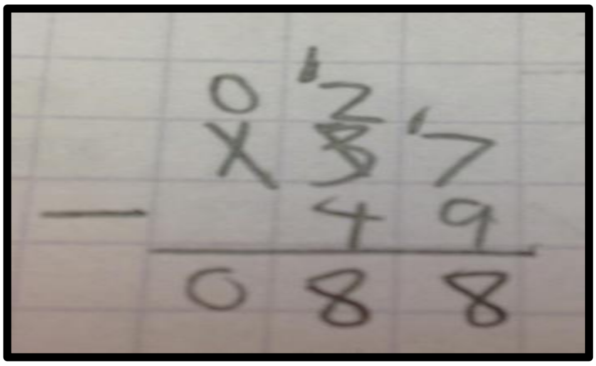

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Multiplication

Definition: Multiplication is the product of two numbers or repeatedly adding the same set of number as many times as the other number. Therefore 3 multiplied by 4, is 4 lots of 3, or 3 added repeatedly 4 times. It is an inverse of division.

Core Skills:

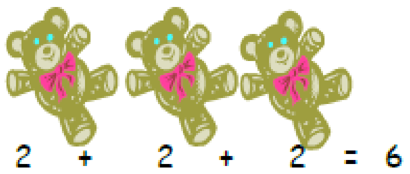
To progress and use efficient methods of multiplication, pupils will need to be able to:

- Count forwards and backwards in multiples e.g. 2's, 4's 10's.
- Use repeated addition
- Rapidly recall multiplication facts
- Partition
- Understand place value
- Multiply and divide by 10, 100 and 1000
- Double and halve
- Use division as the inverse of multiplication.

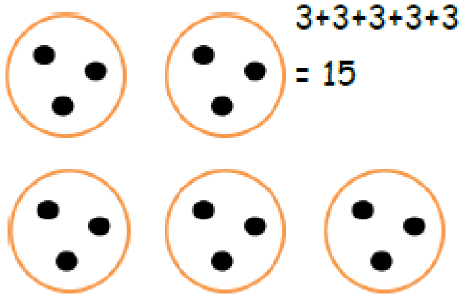
Early Learning

Pupils manipulate and experience a range of resources in real life contexts and play. They are encouraged to solve real life problems.

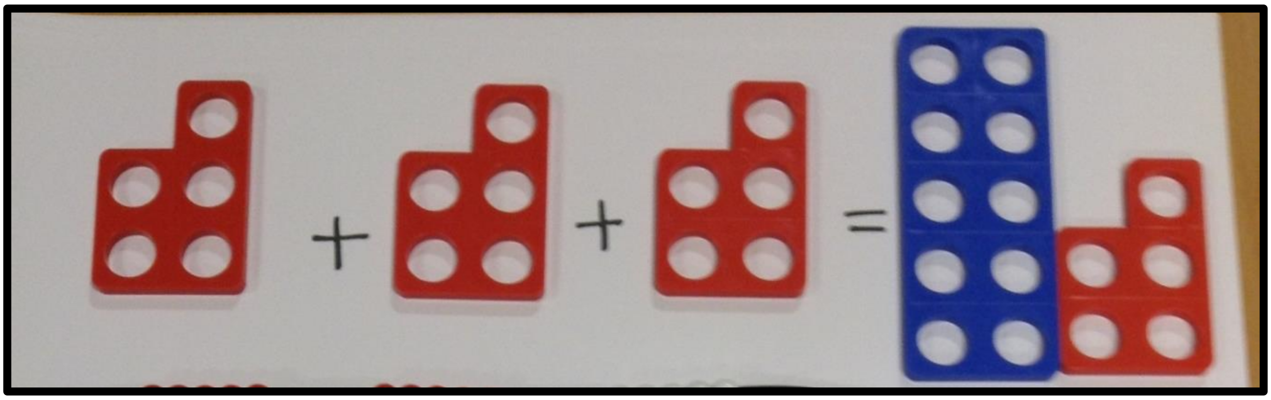
How many legs will 3 teddies have?



There are 3 sweets in one bag. How many sweets are in 5 bags altogether?



They draw pictures and show their mathematical thinking through various representations, e.g. pennies, cubes, bead strings, Numicon.

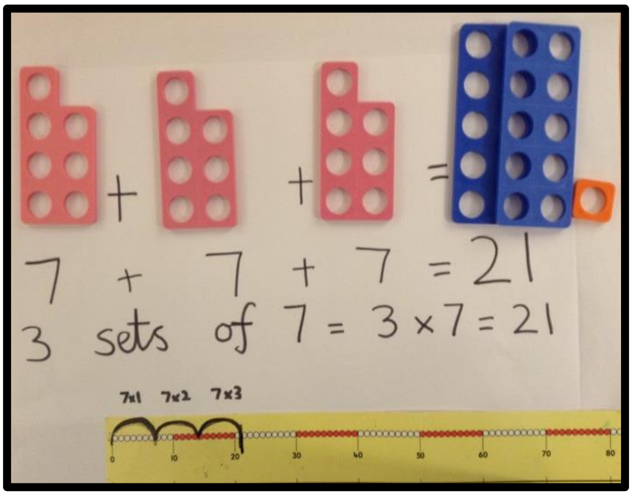


Children recall multiplication facts and develop strategies to use known facts for further calculations.

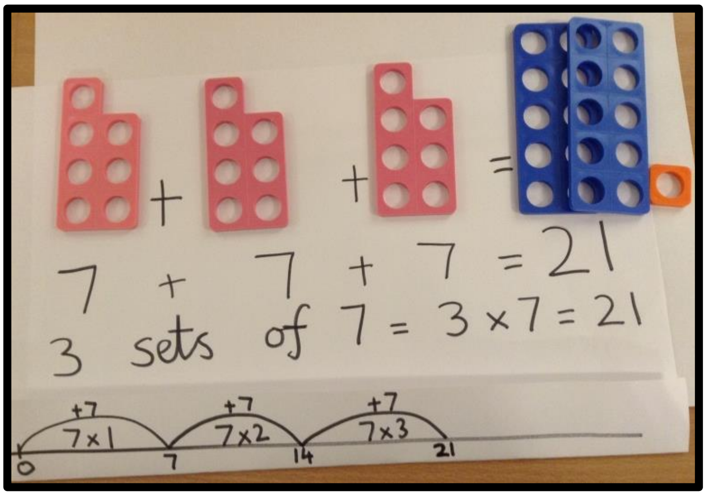
Number lines

Verbalise multiplication as 'lots of' or 'groups of' e.g. 5x3 = "3 lots of 5" to ensure that the children make the connection with repeated addition. Use number lines alongside concrete and pictorial, drawing links with measure and money.

On a marked number line:



On an empty number line:



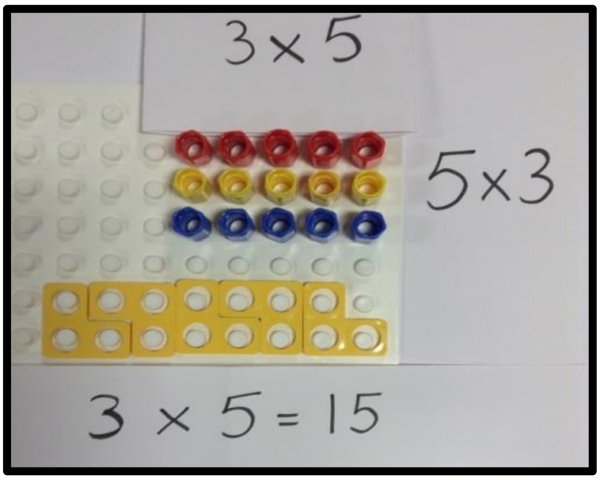
Arrays

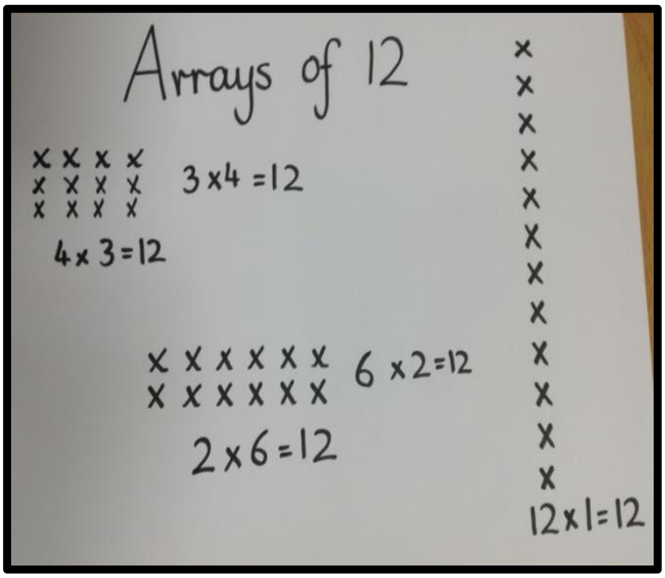
Arrays are taught alongside number lines to help prepare for grid method. Count groups of objects in each row or column to find the product. Give a number of objects to arrange in an array of various dimensions and also give an unknown number of objects to count by arranging in arrays.

3 lots of 5 = 5 groups of 3

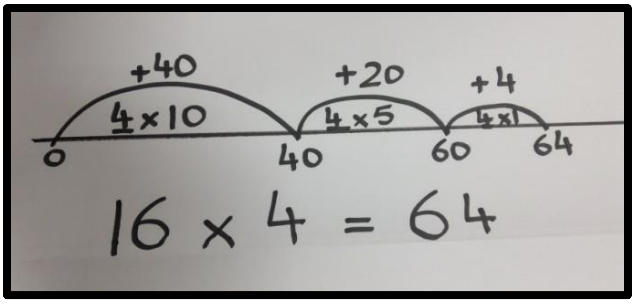
5 x 3 = 3 x 5

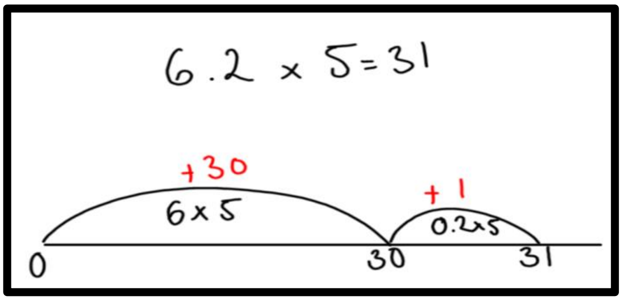
5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = 15





Partitioning using number lines and grids

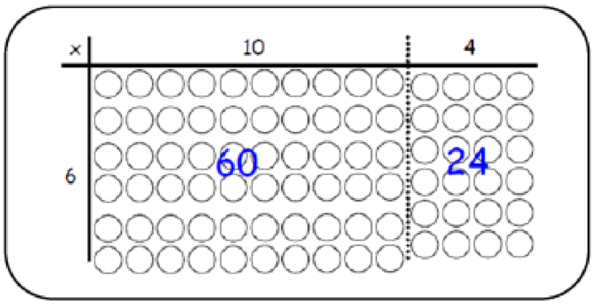




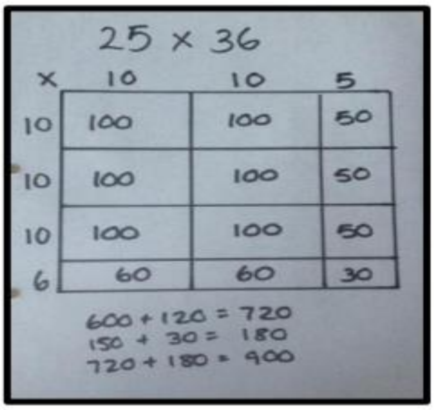
This should be taught at the same time as arrays and can be done using decimals as well.

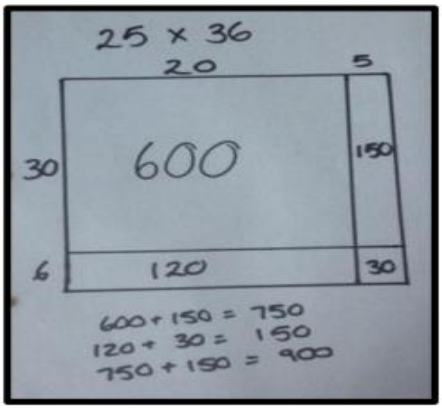
Grid method

Progress to grid method, using increasingly larger numbers making links with arrays.



Multiply two digit numbers by partitioning and keeping it proportional.

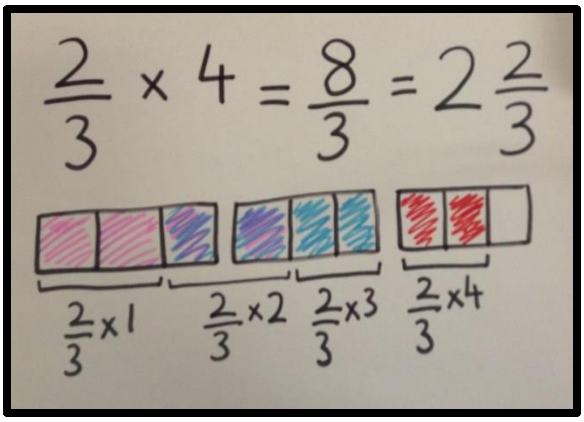


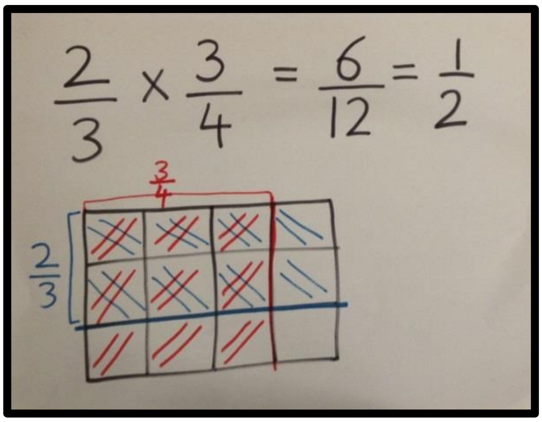


NOTE: when teaching grid method. NEVER say take off 0's and add them back on. Develop fluency skills using known facts e.g. 20 x 30 = 2 x 10 x 3 x 10.

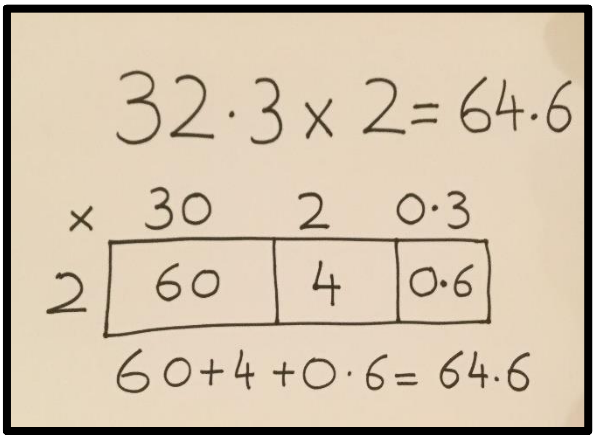
Multiplication of fractions and decimals:

Use of bar model is encouraged to develop understanding:





Multiply with decimals using grid method and partitioning.

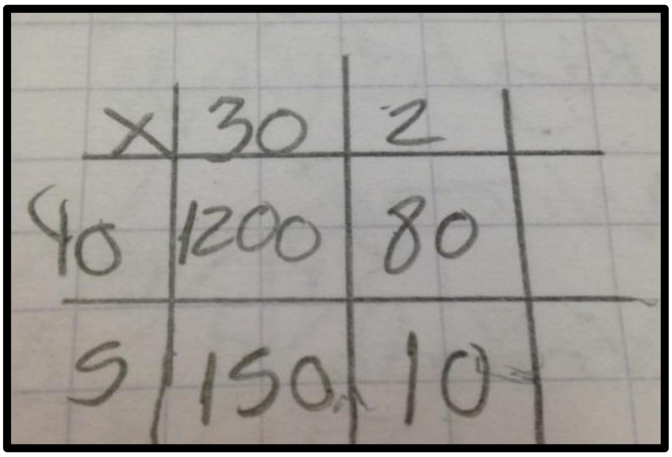


Formal written method

The formal method should only be taught when appropriate. Further examples can be found in the appendix.

Expectations of Downham Market Academy

Pupils continue to use mental methods, number lines and progress to column methods with a greater understanding.



Web Links: https://www.ncetm.org.uk/

Division

Definition: Division is repeated subtraction or splitting into equal parts or groups. It is the result of "fair sharing".

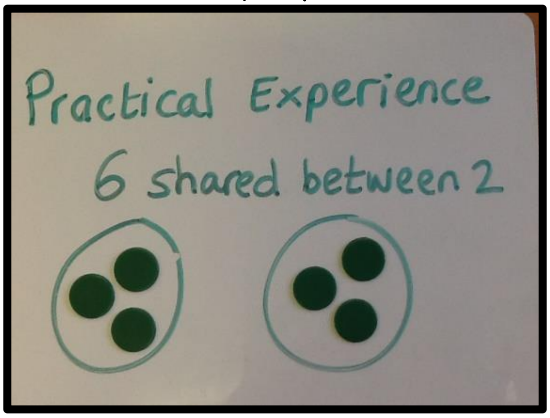
Core Skills:

To progress and use efficient methods of division, pupils will need to be able to:

- Recall multiplication facts
- Understand division as the inverse of multiplication
- Count accurately
- Understand place value
- Multiply and divide by 10, 100 and 1000
- Double and halve

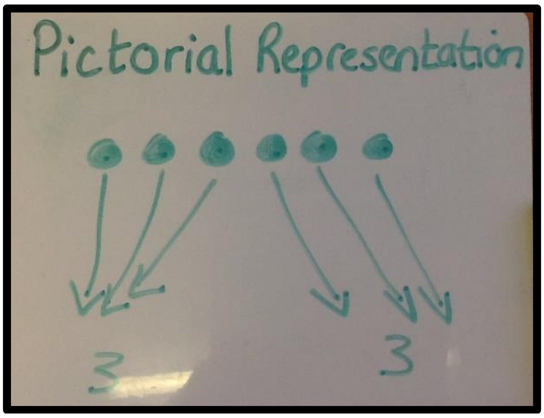
Early Learning

Children use concrete resources to share equally in real life contexts.

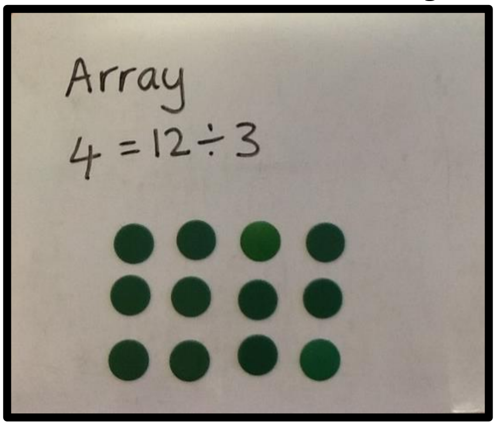


Pictorial Representation

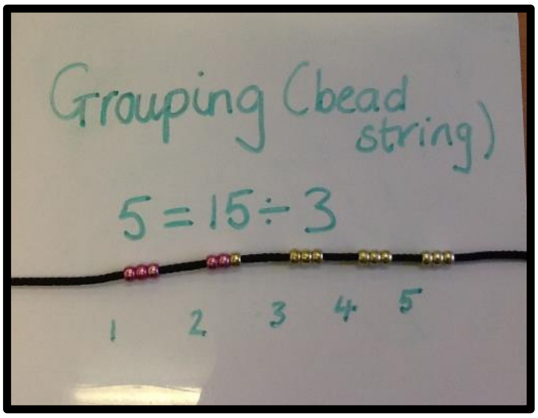
Children share equally using pictures to support their understanding.

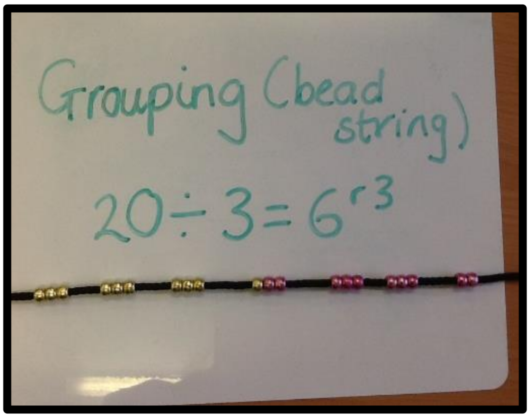


Arrays
Children use concrete and pictorial representations for division calculations (including recognising multiplication as the inverse of division through arrays).

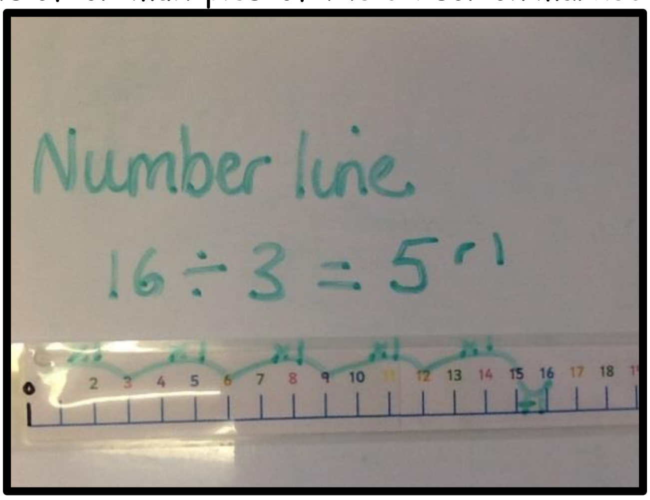


Grouping (bead string)
Children learn that dividing can be done as groups including equal sharing and those with remainders.

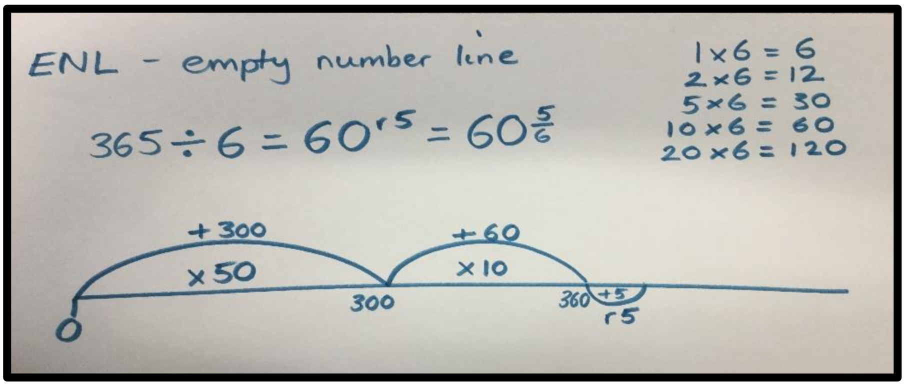




Number lines:
(Counting on or back – addition or subtraction)
Children will count in 'lots of' or 'multiples' of the divisor on marked number lines.



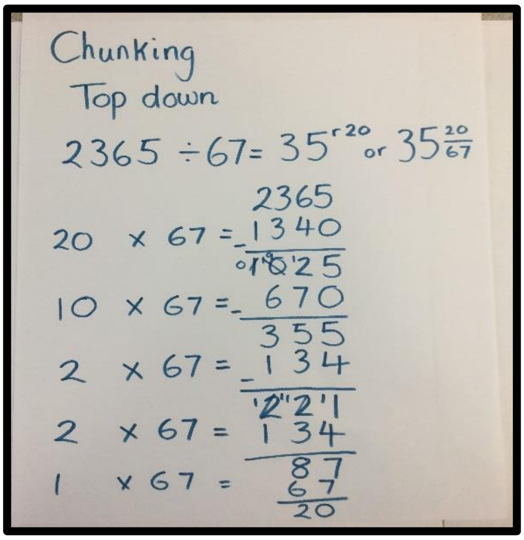
Children will progress on to an empty number line, using the strategies above of counting in 'lots of' or multiples (chunks). To support this method, children list known facts to help them (can be known as coin multiples – 1x/2x/5x/10x/20x/50x/100x)

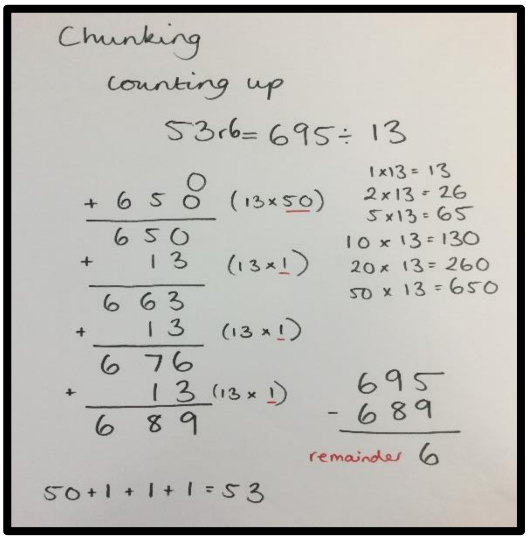


Chunking

Subtract multiples of the divisor from the starting number. Again, the children list the 'coin multiples' to support their chunking.

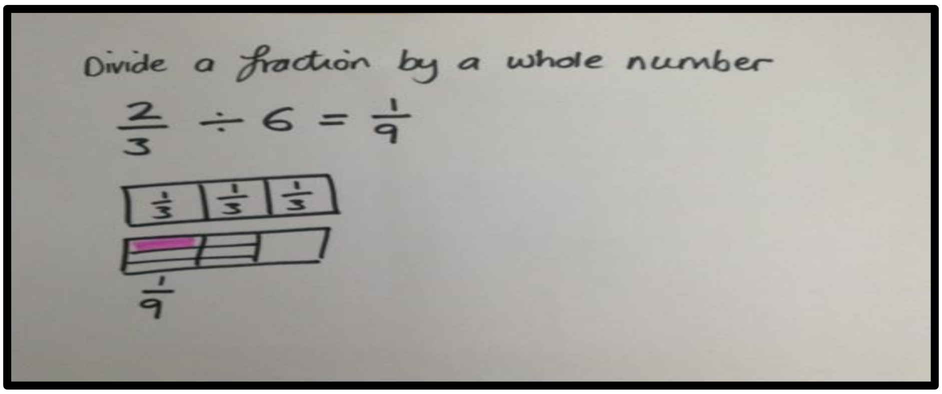
Chunking can be done by subtracting each chunk or by beginning at 0 and adding on each multiple until the total is reached (possibly with a remainder).





Fractions

Use bar models to represent division of fractions.

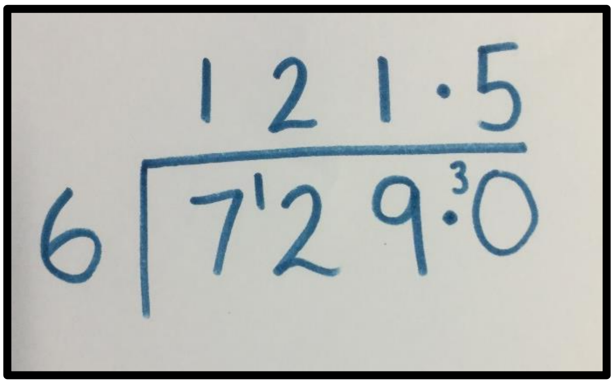


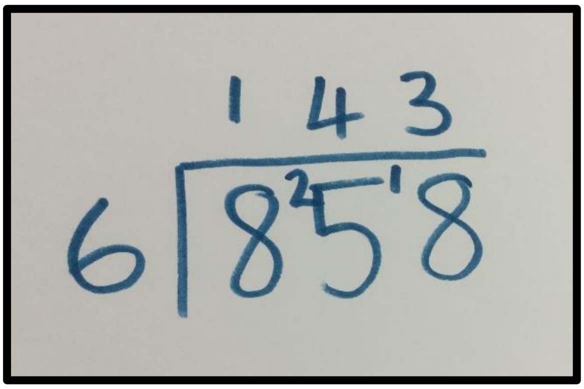
Formal written method

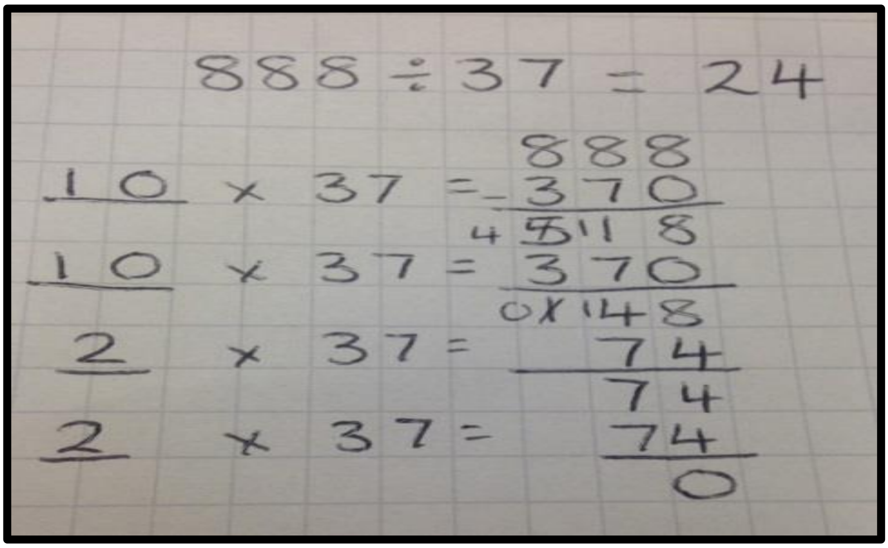
The formal method should only be taught when appropriate. Further examples can be found in the appendix.

Expectations of Downham Market Academy

Pupils continue to use mental methods, number lines and progress to bus stop methods with a greater understanding.







Weblinks: https://www.ncetm.org.uk/

Algebra

What is algebra?

Algebra is a way of thinking and a set of concepts and skills that enable pupils to generalise, model and analyse mathematical situations.

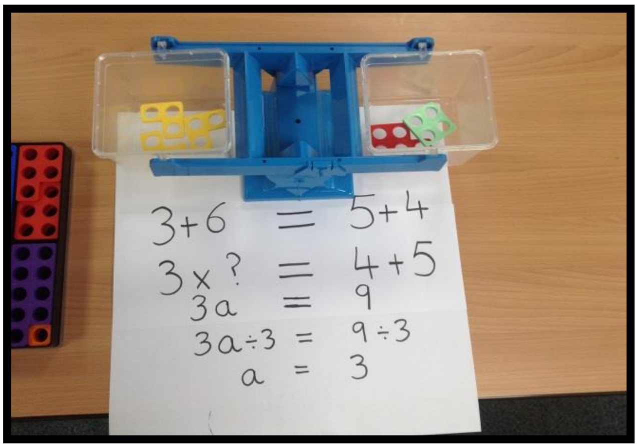
Why is it important?

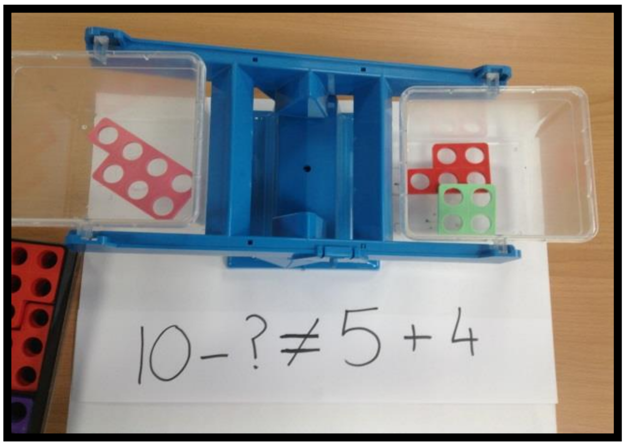
Algebra provides a systematic way to investigate relationships, helping to describe, organise, and understand the world.

Although learning to use algebra makes students powerful problem solvers, these important concepts and skills take time to develop. Understanding of Algebra begins early and should be a focus of mathematics instruction from EYFS through all Key Stages.

Equals sign =

In Maths equality (=) means balance between two sets and inequality (≠) means an imbalance.





Algebra requires children to solve simple equations that involve addition, subtraction, multiplication and division with a deeper understanding of the equality symbol. Using concrete resources to start with, they should be able to explore the equality and inequality of values of numbers.

A helpful pedagogy to use is, '**What's the same and what's different** on both sides of the equation?' There must be an opportunity to experience some examples of inequality to appreciate equality in a greater sense.

Generalisation

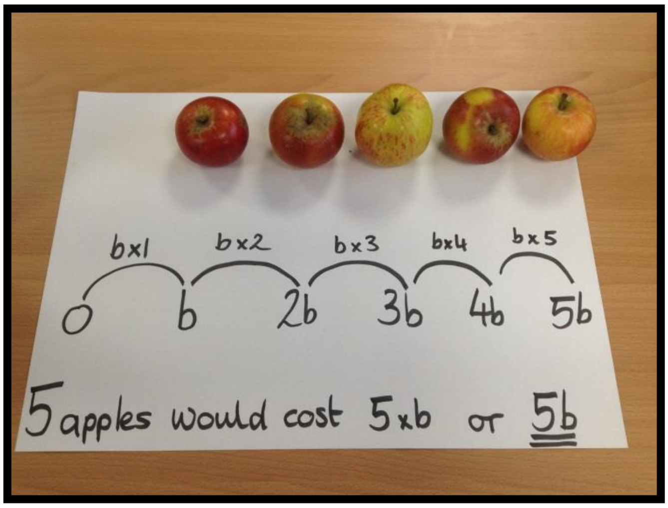
Children should be encouraged to complete the sequence to the n^{th} term by generalising.

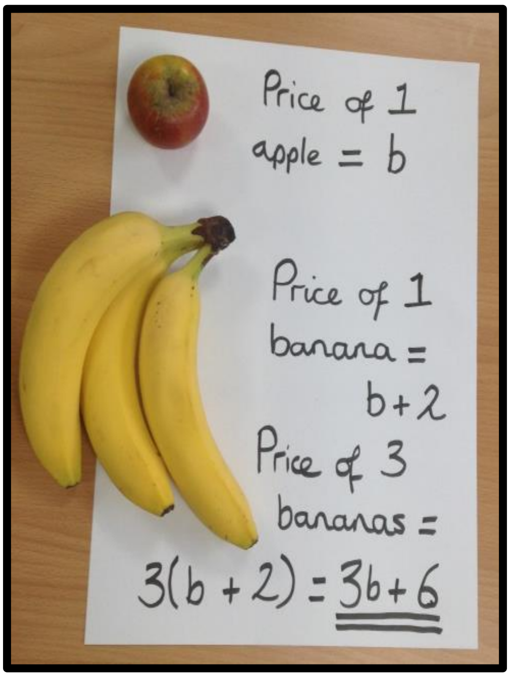
e.g. 5, 8, 11, 14, 17, ___ so the nth term = 3n + 2

Children find the unknown or the missing numbers in all areas of calculations.

e.g. If each banana costs 2p more than an apple, what is the cost of 3 bananas?

What is the cost of 5 apples?

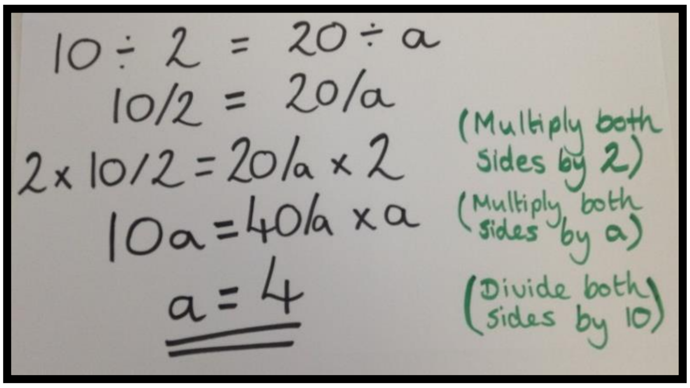




Pupils should be encouraged to make links e.g.

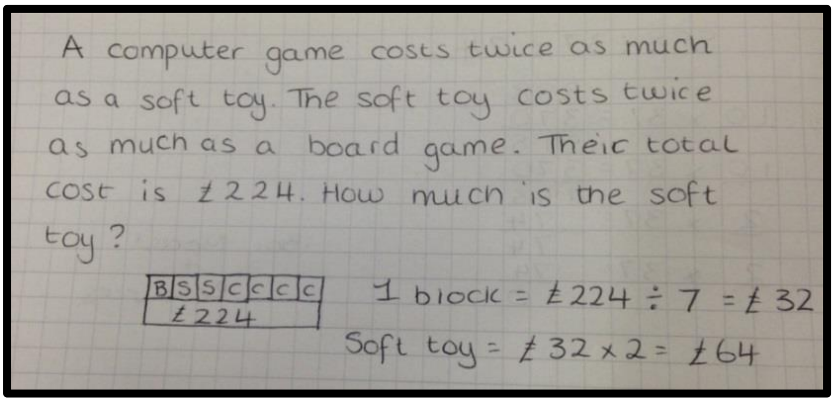
2 x 10 + 3 x 10 = 5 x 10

Find the missing numbers to solve problems e.g.



Expectations of Downham Market Academy

Children will continue to use the bar model for visualisation where appropriate.



What does fluency, reasoning and problem solving look like in solving calculation questions?

These are the three aims from the 2014 Mathematics National Curriculum which are to ensure all pupils:

- become **fluent** in the fundamentals of mathematics, through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The 2014 mathematics curriculum states that 'Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas… (*all*) pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.'

Examples of fluency, reasoning and problem solving:

2 x 7 = 14

Starting with this problem, pupils who demonstrate good fluency, reasoning and problem solving skills are able to use this fact to create others such as:

7x 2= 14	7 x 2 = 14 x 1	2 x 7 = (2 x 10) – (3 x 2)
14 ÷ 2 = 7	(2x3+1) x 2 = 10 + 4	0.2 x 0.7 = 1.4
14 ÷ 7 = 2	4 x 3.5 = 14	7 x 2 = 7 + 7
2 x 70 = 140	14 x 7 ≠ 2	14 = 7 x 2
20 x 70 = 1400	2 x 7 = 2+2+2+2+2+2+2	2000 x 7 = 14000

MASTERY

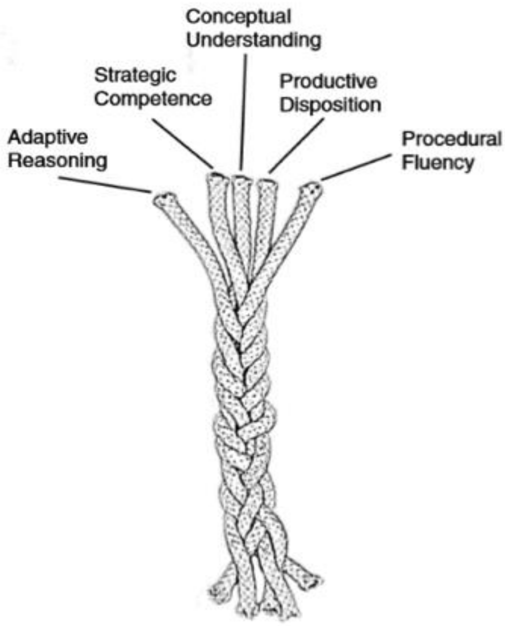
Mastery of maths means a deep, long-term, secure and adaptable understanding of the subject. Among the by-products of developing mastery, and to a degree part of the process, are a number of elements:

- fluency (rapid and accurate recall and application of facts and concepts)
- a growing confidence to reason mathematically
- the ability to problem solve in Maths, to conjecture and to test hypotheses.

Mastery of maths, which should build gradually as a child goes through school, is a tool for life, and immeasurably more valuable than the short term ability to answer questions in tests or exams.

Significantly, some of the implications of adopting mastery approaches to teaching maths are relatively new. One of these is the move away from labelling pupils as 'high ability' or 'low ability' and giving them different tasks. Another is the approach, especially in the early primary years, of reducing the amount of mathematical topics handled in class, but taking longer over each one, so that early understanding is cemented more sustainably.

The interwoven and interdependent nature of the five essential aspects are powerfully captured by the image: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and productive disposition.



Adding it up (Kilpatrick, Swafford and Findell, 2001, p.115)

Concrete Resources

Resources must be used to support learning throughout all key stages. Use of concrete, pictorial and abstract methods should be considered good practice in all learning environments. A full list of resources can be found [here](#).

Expectations of Downham Market Academy

Concrete resources will be used and encouraged to aid and deepen understanding.

Useful IT Resources

Curriculum, Pedagogy and Planning

The National Curriculum: https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study#history

My Maths Planning and Resources: www.mymaths.co.uk

NCETM: https://www.ncetm.org.uk/

STEM: https://www.stem.org.uk/

Problem Solving

NRich site for problem solving: http://nrich.maths.org/frontpage

The Mathematics Shed: http://www.mathematicshed.com/

Games and Activities

Activities for all year groups from YR to Y13: www.ixl.com

Go Gordons Interactive Maths: http://www.wldps.com/gordons/

Top Marks Games: http://www.topmarks.co.uk/maths-games/5-7-years/counting

Algebra tiles: http://technology.cpm.org/general/tiles/

Interactive Cuisenaire rods: https://nrich.maths.org/4348

Interactive bar modelling:
http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20_tool.html

Problem solving activities/Maths games: http://www.transum.org/Software/

Starters, Practice questions, Videos, Revision: https://corbettmaths.com/

Frequently Asked Questions

Why do we need a cluster calculation policy?

A cluster calculation policy will allow for a fluent movement from primary school into secondary school. As the policy runs from a group of primary schools into Downham Market Academy, it will allow for the children to experience a continuous mathematical journey as DMA will be able to continue with what has been taught, and not re-teach and undo previous methods. It gives DMA an insight into what has gone before.

What if my child isn't going to attend Downham Market Academy?

Regardless of whether or not your child is going to attend DMA, your child will benefit from a very succinct calculation policy. The policy has been written by many Maths Subject Leaders across the cluster, allowing for expertise to be shared and ideas to be made concise and clear. The cluster calculations policy is something that is becoming increasingly common place across Norfolk, and most are very similar; again allowing for consistency to be maintained.

Why are there no year groups within the policy?

It is the belief across the cluster and Norfolk that placing a method within a year group is not beneficial. The child needs to progress through the methods at their own pace. This allows for children to accelerate through it, or slow down if this is necessary. However, deeper understanding should be aimed for all pupils through the mastery approach.

Why are there so many resources in the policy?

Research has proven the benefit of using Concrete, (resources such as cubes and numicon) Pictorial, (methods that show a picture of the resources, such as a number line) and Abstract methods (digits and symbols) within the maths class. They allow for the mastery of a method much quicker than merely teaching the children a procedure (such as a column method).

Why are column methods not introduced earlier?

The Norfolk Maths Team has undertaken much research on the use of column methods and whether or not the introduction of them earlier in the school is beneficial (please see Appendix 2).

Glossary

Abstract	Written equations calculated without the support of concrete objects or pictorial representations.
Add +	Putting two or more sets or numbers together and counting them altogether to find a total. Related vocabulary: more, sum, plus, increase, count on. E.g. 4 + 8 = 12
Algebra	A symbol representing a number; letters are used to represent unknown numbers.
Array	An ordered collection of counters, numbers etc in rows and columns
Commutative	In addition and multiplication the numbers can be represented in any order: E.g. a x b = b x a; a+ b = b + a
Concrete objects	Objects that can be handled to support understanding of mathematical concepts.
Conjecture	An educated guess which has yet to be proven.
Digit	Symbols of the number system: 0,1,2,3,4,5,6,7,8,9: E.g. 24 is a 2-digit number
Divide ÷	Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found.
Equal =	Means *balance* or is equal to, having the same value. E,g 7 + 2 = 9
Equation	A mathematical statement expressing that two things are equal. 4 + 5 = 9
Even number	An integer that is divisible by two with no remainder.
Fluency	To have a combination of conceptual understanding, procedural fluency and knowledge of facts appropriate to the stage of development in order to solve mathematical problems efficiently, confidently and accurately.
Generalise	A statement that applies to all relevant cases. E.g. an odd number added to an even number will always give an odd number as an answer.
Inequality	the relation between two expressions that are not equal, employing a sign such as ≠ 'not equal to', > 'greater than', or < 'less than'
Integer	Any of the positive or negative numbers and zero.
Inverse	To reverse something or undo the previous

	calculation. Multiplication is the inverse of division. E.g. 5 x 6 ÷6 = 5 Addition is the inverse of subtraction. E.g. 5 + 6 – 6 = 5
Multiply x	Scaling one number by another. Related vocabulary: Lots of, groups of, sets of, repeated addition, times. E.g. 3 x 4 = 12 = 3 + 3+ 3+ 3
Odd number	An integer that has a remainder of one when divided by 2.
Operation	Multiplication, division, subtraction and addition are all operations.
Pedagogy	The teaching method used. E.g. Open questioning, generalising, looking for patterns etc.
Pictorial representations	Pictures and images to represent a mathematical concept.
Place value	The numerical value of a digit that relates to its position within a number.
Reasoning	The ability to justify answers through generalisation and conjecturing relationships. Prove using mathematical language.
Subtract	Taking one number away from another or finding the difference when comparing numbers. Related vocabulary; decrease, take away, count back, less.